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Estimation and Identification Process Using an Exponential Forgetting Factor

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Abstract. System identification and parameter estimation are important to obtain information from systems which are difficult to model and that are usually presented as Black- Box models. This work presents a point to point parameter estimation of a generalized non-deterministic system, whose results are variable through time, by using an exponential Forgetting Factor (FF). An average approximation is used as base to add an exponential FF to modify and improve the average results, without increasing the computational cost considerably. A comparison of the results applying the Least Square Method (LSM), the Recursive Least Square (RLS) and FF is presented using a signal for tracking a simple trajectory to prove the performance of the proposed method. As conclusion, it is obtained an online estimation for a non-deterministic signal without needing a previous training or Knowledge Base (KB).

Keywords: parameter estimation, system identification, exponential forgetting factor, least square method.

1 Introduction

A system is an arrangement of different elements interacting together to accomplish an objective [1, 2]. There are, for example, biological, physical, chemical systems or the combination of them. For their analysis, representative models are created, whose complexity depends on the number of considered details. To begin an analysis is necessary to identify the known characteristics, which generally are the inputs and outputs seen through measurable signals. Finally, a mathematical expression could be determined based on the description of the behavior and the relation between the variables and parameters.

Models created considering the involved physical phenomena are difficult to obtain because it is necessary to know what happens in the system, how it interacts with its environment and what its boundaries are. A Black-Box (BB) description, shown in Fig.1, is useful when only the input and output signals are known, and then the internal dynamic is obtained by parameter estimation techniques [3, 4, 5, 6].

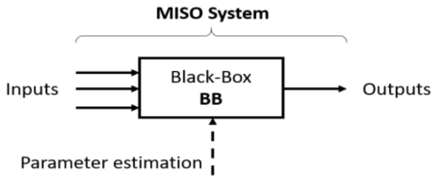

Fig. 1. Black-Box (BB) model for a Multiple Input Single Output (MISO) system.

System Identification (SI) and Parameter Estimation (PE) are related process, due to the first one depends on the second. First it is necessary to determine, according to the known signals, the parameters that describe the dynamic system. These parameters will allow to identify, from an input signal, an output identified signal; which is supposed to describe a reference system in an approximated form.

Different techniques have been developed and studied, such as a Fuzzy Logic (FL), Lyapunov, Sliding Modes (SM) and Intelligent Systems (IS) or their combinations integrating Hybrid Estimation- Identification (HE-I) systems.

According to [7] and [8], hybrid methods not only require a high number of operations but also of computing time. In order to reduce these magnitudes, the mathematical expectation is used. Nevertheless, a non-stationary system requiring a point to point approximations will not be modeled adequately considering only an average estimation because new parameters are necessary for each variation in the reference, and for these cases the statistical information is not useful. So that, a Forgetting Factor (FF) must be used in any method to reduce the convergence error between the reference and the identification results deleting unnecessary past information.

Within the stat of the art, FF has been represented through constant coefficients, linear functions, and exponential functions considering the Euler number, by intervals or with a specific sample time [9]; in almost all cases the FF is used to improve another algorithm [10, 11, 12]. In [13] is mentioned that FF must be between 0 and 1, where close to 0 values discard the information faster than those closer to 1.

In fact, real systems are variable, meaning their characteristic parameters too, creating the necessity developing algorithms that allow the calculation of parameters under non-constant conditions within time restrictions. Thus, for variable systems, it is necessary to determine also a variable FF to obtain an adaptive response.

In the present paper the effectiveness of an Exponential Forgetting Factor (EFF) applied to an average estimation is proved by implementing following steps: first, an equivalent Multiple Input Single Output (MISO) system seen as a BB with variable parameters; then, it is used a recursive description to obtain an online estimation, after that, the error between the input and output is used to determine the necessary convergence to reach a reference using the magnitude and sign properties as arguments in the last estimation stage and finally, obtaining values are applied into BB model to complete the system identification process. The method is implemented in a signal tracking problem and is compared to the Least Square Method (LSM) and the Recursive Least Square (RLS) to identify their advantages.

2 Average Estimation

Considering an Autoregressive Moving Average (ARMA) of first order with a single delay, due to the characteristics and description presented in [14], it is possible to estimate the unknown parameters $\hat{A}_t$ from the reference signal y_t and the inputs of a system x_t, as shown in Fig. 2, using the average expected value as in (1), where T and $+$ are the transpose and inverse operations, respectively. Then, the identified signal is obtained using the estimated parameters as in (2).

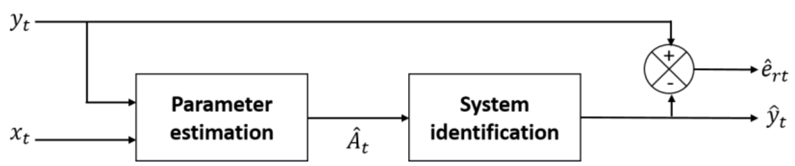

Fig. 2. Parameter estimation and system identification from a first order ARMA model.

$$\hat{A}_t = E\{y_t x_t^T\}(E\{x_t x_t^T\})^+, \tag{1}$$

$$\hat{y}_t = \hat{A}_t x_t. \tag{2}$$

Comparing the reference and the identified signals obtaining the identification error $\hat{e}_{\mathrm{rt}} = \hat{y}_t - y_t$. Based on its functional J_{et} an average parameter estimation gave a zero error only in the infinite, point in which the estimation is considered optimal [15].

3 Recursive Parameters Estimation

When using the concept of Functional Error (FE), the recursive description (3) of the estimated parameters is obtained.

$$\hat{A}_t = t^{-1} y_t x^T Q_t^+ + t^{-1}(t-1)\hat{A}_{t-1} Q_{t-1} Q_t^+, \tag{3}$$

where $Q_t = t^{-1} x_t x^T + t^{-1}(t-1) Q_{t-1}$ and then applying (3) in (2) the recursive description of the identified signal is obtained. An explained description of (2) is presented in [14].

At this point, an average estimation, useful for systems with small parameters variations. The problem with those having a non-deterministic behavior is that we do not know if perturbations breaking the stationary condition, or leading the system out of their boundaries, will be presented.

4 EFF Estimation

From the average estimation, its low computational complexity is a remarkable advantage. In cases when a unique parameter is used in combination with an adequate FF [16] it presents a better performance compared to other hybrid versions [7,17].

Within the state of the art, FF is implemented to modify the covariance from a system state, but the descriptions are complex due to the number of operations, which increase in order of the parameter matrix. Each FF affects with different intensity the estimation results, having the exponential kind the most accepted, due to the past information tends to lose weight through the evolution of the system following the tendency of the exponential function (4) [18], whose base is the number e, and C and arg the coefficient and argument that affects its behavior.

$$\exp = Ce^{arg} \tag{4}$$

Two functions presented in different calculations, and that are not extensively used to modify the Forgetting Factor (FF), are the *sign* (5) and the absolute (5). Each of them gives different properties from magnitudes without increasing considerably the complexity.

$$|x| = \begin{cases} x & if\ x \geq 0, \\ -x & if\ x < 0, \end{cases} \tag{5}$$

$$sgn(x) = \begin{cases} 1 & if\ x > 0, \\ 0 & if\ x = 0, \\ -1 & if\ x < 0. \end{cases} \tag{6}$$

A comparison of both functions is developed to determine which is more useful for the error correction in the estimation-identification process. Fig. 3 shows the results.

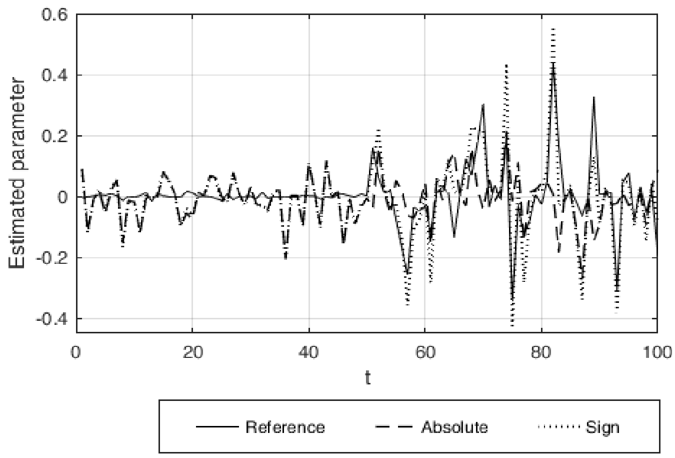

Fig. 3. Comparison of the sign and absolute function in the approximation of a signal.

As in shown in Fig. 3, comparing both functions to approximate a reference with different magnitudes of variations, the sign function performs better, so this is the one selected. It is also possible to see that when the reference tends to zero, both approximations are similar, obtaining a more remarkable variation when the error is different from zero.

Considering the error identification as an innovation process and the principal argument for the exponential correction factor, as well as the characteristics given by the sign function, it is proposed in [19] the use of the Exponential Forgetting Factor (EFF) (7).

$$EFF_t = sgn(\hat{A}_t)e^{sgn(\hat{A}_t)e_{rt}}. \quad (7)$$

This factor is applied to modify the first average estimation and obtaining a second estimation as indicated in (8)

$$\hat{\hat{A}}_t = \hat{A}_t + EFF_t - sgn(\hat{A}_t). \quad (8)$$

To obtain the final identified output is necessary to apply (8) in (2); obtaining a corrected estimation in two complete iterations. The block diagram in Fig. 4 summarizes the procedure, where the dotted lines indicate the second estimation stage.

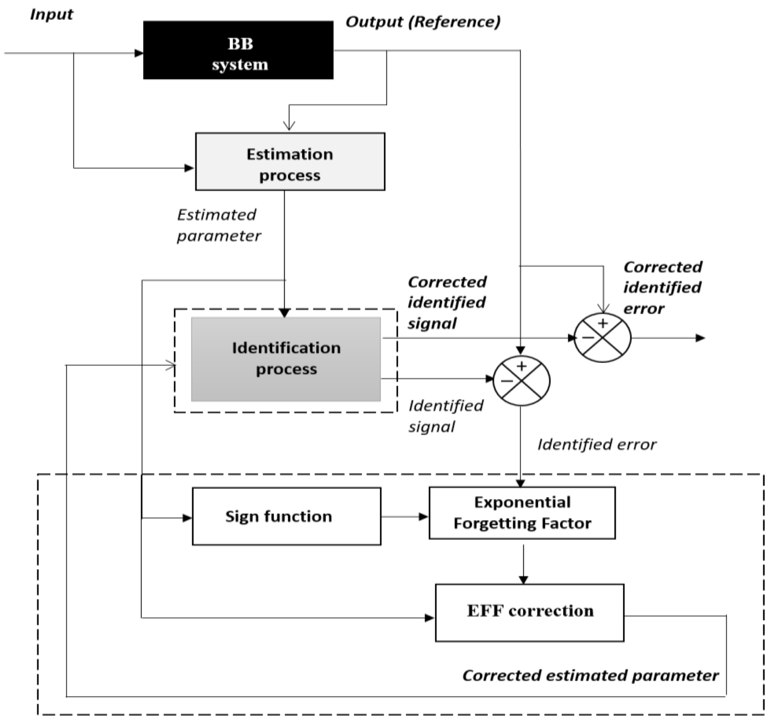

Fig. 4. Estimation-Identification block diagram, with two stages of correction using the sign function and the Exponential Forgetting Factor (EFF).

5 Application Example

The estimation-identification process analyzed in time is focused on the shape of a signal. To visualize the effectiveness of the EFF correction, the method is applied in a simple tracking trajectory problem.

According to [20], the initial trajectory assigned to a flight could be adjusted through time to optimize the path with the objective of avoid accidents and to take advantage of the wind flow [21]. Searching benefits in security, reduction of the fuel consumption and environment pollution.

Being the reference signal a trajectory from one point to another given in latitude and longitudinal coordinates, first the Recursive Least Square (RLS) and the Least Square Methods (LSM) are compared.

To implement these first two methods is necessary to determine the longitude of the filter and their initial parameter values. Longitude could be between *1* to *n*, being *n* the biggest value, and the initial parameters between *0* and *1*.

In Fig. 5 different filters size and their execution time are presented. This data was obtained considering the input as a random signal and output signal from the reference trajectory.

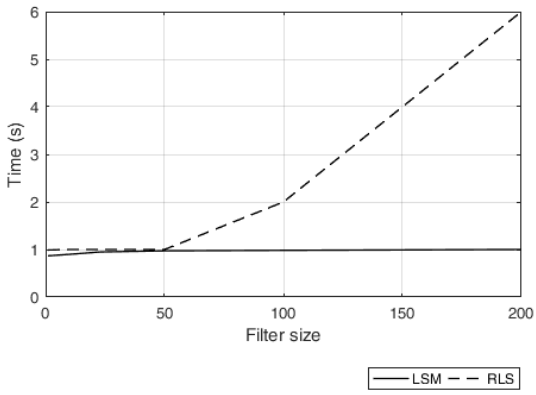

Fig. 5. Comparison in execution time between using the Least Square Method (LSM) and the Recursive Least Square (RLS).

From Fig. 5 is easy to appreciate that an increment in the filter size implies increasing the execution time. Thus, a second is performed to identify which filter, LSM or RLS methods, approximates better the reference and with what size. Fig. 6 shows these performances considering filters of size *1*, *23* and *50*.

The *23* size LSM was the best adapted to the reference, while any size of the RLS method could not approximate the trajectory.

From previous results, it is decided to use the LMS (23) as reference to compare the correction through the EFF estimation. Fig. 8 shows the approximation to the reference comparison, and Fig. 9 the error obtained with each method, where the LSM has an approximate of *2.5%* error while the EFF *6%*.

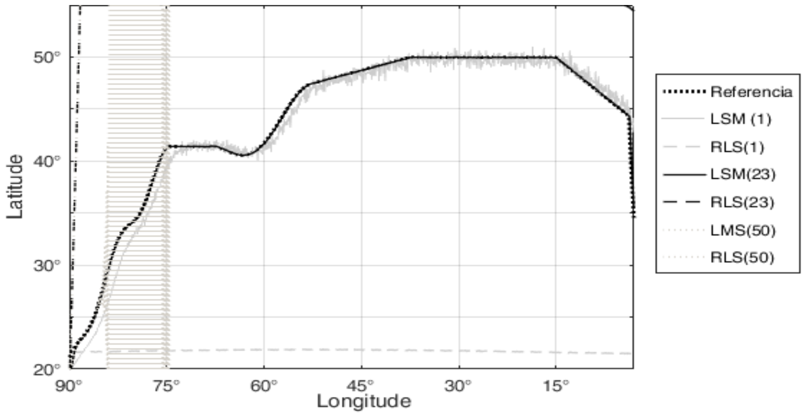

Fig. 6. Comparison of the LSM and RLS performance for different filter sizes.

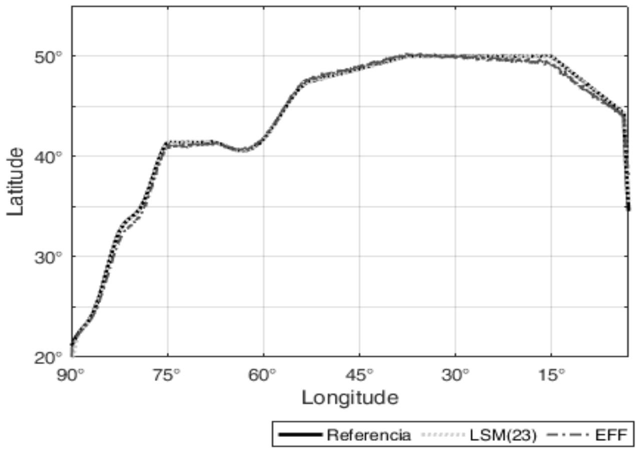

Fig. 7. Performance in the tracking trajectory problem using the LSM and the EFF methods.

In addition to this, another comparison is performed, but now using the CPU-time from the Matlab® interface to measure how much time they take to accomplish the estimation. Fig. 9 presents the results, where the EFF estimation takes less time than the LSM method.

6 Conclusions

With the development of the present paper, it was possible to compare a parameter estimation - system identification process by using an Exponential Forgetting Factor (EFF), reducing the influence of past data in comparison to the traditional Least Square Method (LSM) and the Recursive Last Square (RLS).

The identification obtained with the LSM and the EFF was similar and better than that one obtained with the RLS, for the considered signal reference. When using the adequate size filter for the LSM the error tends to zero, nevertheless, when using the EFF it is not necessary to define initial parameters and its execution time is *75*%, approximately, lower than the one LSM presents.

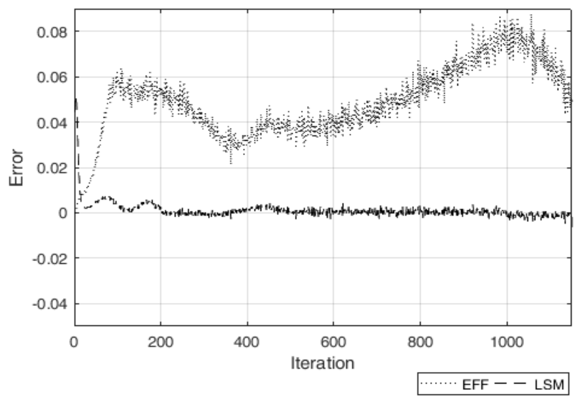

Fig. 8. Error obtained when LSM and EFF methods are applied to identify a reference.

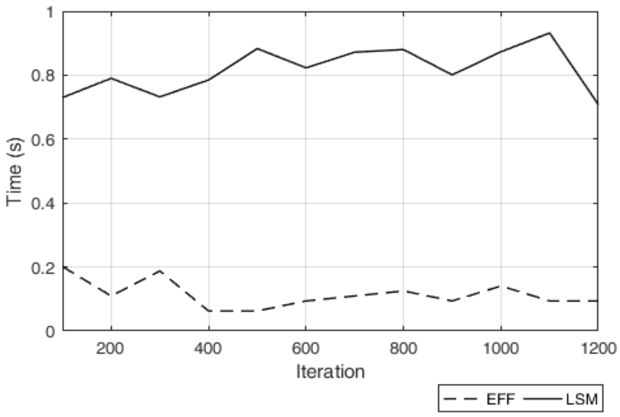

Fig. 9. Execution time taken from the *cputime* given by Matlab for the tracking trajectory problem, measured for different number of iterations and comparing the EFF and the LSM.

It is necessary to compare more Forgetting Factor (FF) configurations to determine which is the best, but for now the combination of the sign function of the estimated parameter, in average, and the error magnitude, as presented in this work, gives useful results when time restrictions are presented.

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